WELCOME GUYS!

	JAN	Feb	mar
A2 (5–6pm)	S1	P3	P3

NON - AS - SCHOOL (A1) ⟶ P1 / P3

DO NOT TAKE S1 THIS SESSION

	Feb	Mar
AS (4–5pm)	P1	P1
A2 (5–6 pm)	P3	P3

2000 – 2019 (P6)
2020 – on (P5)

(S1) (50 Marks) (20%) (1h 15m) (7 Questions)

SUPER EASY PAPER IF YOU KNOW HOW TO READ ENGLISH AND COMPREHEND A STORY.

(S1)

READING & UNDERSTANDING STORY	MATHS
80% READING TIME	20% SOLVING

HIGH PERCENTILE PAPER : 43 — 46 out of 50.

PROBABILITY

Study of chance:

RAIN	NOT RAIN
30%	70%

$P(\text{RAIN}) = \frac{30}{100} = 0.3$

$P(\text{NOT RAIN}) = \frac{70}{100} = 0.7$

(Impossible) 0 —— 50–50 —— 1 (CERTAIN)

$$P(\text{event}) = \frac{\text{Favorable outcomes}}{\text{Total Outcomes}}.$$

SAMPLE SPACE (LIST OF ALL POSSIBLE OUTCOMES IN AN EXPERIMENT)

$\boxed{1}$ TOSS A COIN : H, T

$\boxed{2}$ ROLL A FAIR DICE (DIE) : 1, 2, 3, 4, 5, 6

$\boxed{3}$ TOSS TWO COINS : HH, HT, TH, TT

$$P(\text{a head and a tail}) = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{first head and second tail}) = \frac{1}{4}$$

$\boxed{4}$ TOSS A COIN AND ROLL A DICE TOGETHER.

H1	T1
(a) H2	T2
H3	T3
(a) H4	T4
H5	T5
(a) H6	T6

Total Sample Space = 12

(a) P(head with even number)

$$= \frac{3}{12} = \frac{1}{4}$$

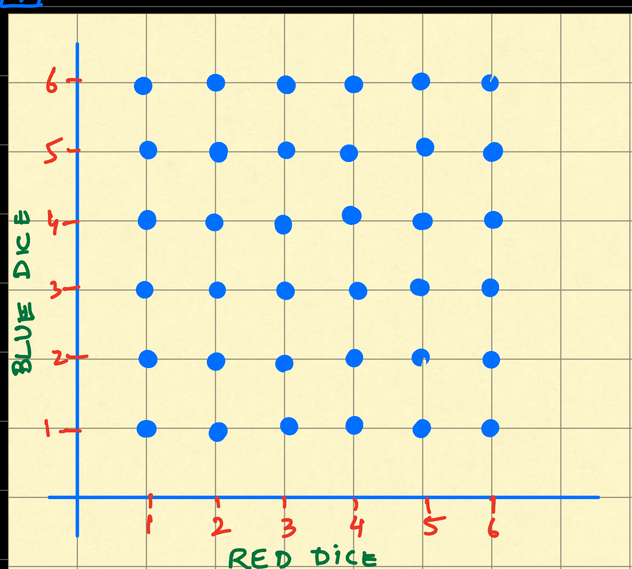
(b) P(Prime number with tail)

$$= \frac{3}{12} = \frac{1}{4}$$

Prime?

Exact 2 factor

$\boxed{\text{Eg}}$ ROLL TWO FAIR DICES:



(a) P(both same scores)

$$= \frac{6}{36} = \frac{1}{6}$$

(b) P(sum is exactly 10)

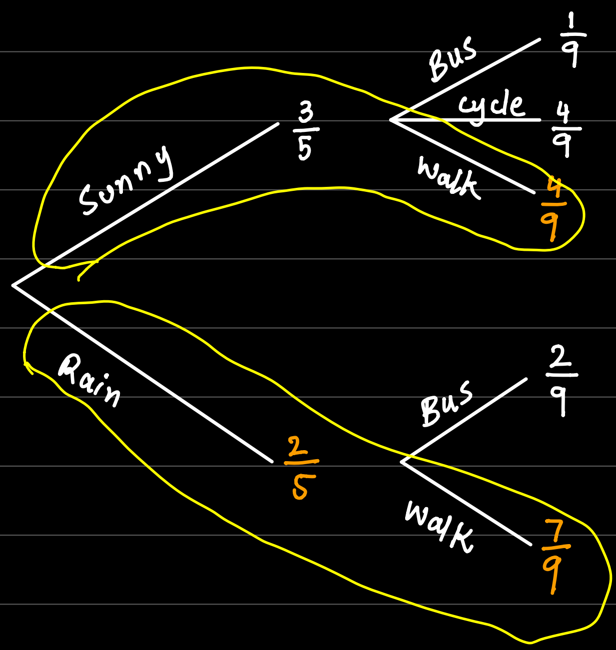
$$= \frac{3}{36} = \frac{1}{12}$$

(c) P(product is less than 8)

$$= \frac{14}{36} = \frac{7}{18}$$

Sample space Total = 36.

TREE DIAGRAMS



RULES

1) SUM OF ALL BRANCHES STARTING FROM SAME POINT = 1

2) BRANCH TOTAL = MULTIPLY.

3) MORE THAN ONE BRANCH

ADD INDIVIDUAL BRANCH TOTAL.

(a) P(sunny and cycle) $= \underbrace{\frac{3}{5} \times \frac{4}{9}}^{\text{S and C}} = \frac{4}{15}$

(b) P(walk) $= \overbrace{\left(\frac{3}{5} \times \frac{4}{9}\right)}^{\text{S and W}} \overbrace{+}^{\text{OR}} \overbrace{\left(\frac{2}{5} \times \frac{7}{9}\right)}^{\text{R and W}}$

$\frac{12}{45} + \frac{14}{45}$

$= \frac{26}{45}$

AND means MULTIPLY
OR means ADD

CONDITIONAL PROBABILITY

GIVEN THAT THE SECOND EVENT HAS ALREADY HAPPENED, FIND PROBABILITY OF FIRST EVENT.

A = FIRST EVENT B = SECOND EVENT.

$$P(\underbrace{A}_{\text{Find}} \,\underbrace{|}_{\text{given that}}\, \underbrace{B}_{\text{has happened.}}) = \frac{P(A \text{ and } B \text{ together})}{P(B)}$$

(d) Given that Ali went to school by walking, Find the probability that it was raining?.

FIRST Event
SUNNY
RAIN ?

SECOND EVENT
BUS
WALK ✓
CYCLE .

$$P(\underbrace{\text{Rain}}_{\text{Find}} \,|\, \underbrace{\text{walk}}_{\text{happened}}) = \frac{P(\text{Rain and walk})}{P(\text{walk})}$$

$$= \frac{\left(\frac{2}{5} \times \frac{7}{9}\right)}{\left(\frac{3}{5} \times \frac{4}{9}\right) + \left(\frac{2}{5} \times \frac{7}{9}\right)}$$

$$= \frac{\frac{14}{45}}{\frac{26}{45}}$$

$$= \frac{14}{26}$$

$$= \frac{7}{13}$$

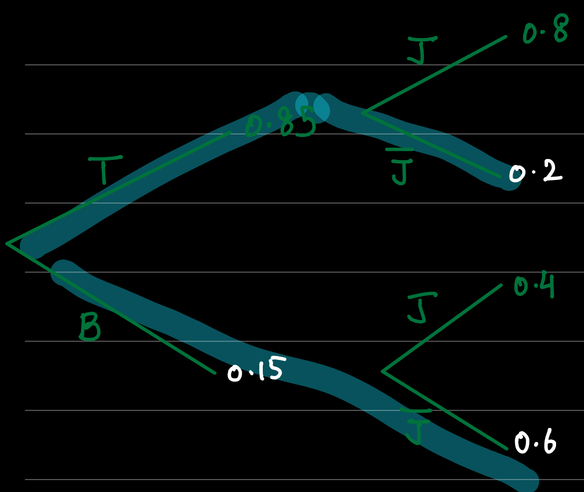
80% Reading Time, 20% Solving Time.

10 Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

(i) Draw a fully labelled tree diagram to show this information. [2]

Second event — First event.

(ii) Given that Maria did **not** have jam for breakfast, find the probability that she had toast. [4]



$\bar{J}$ = No Jam

$$P(T \mid \bar{J}) = \frac{P(T \text{ and } \bar{J})}{P(\bar{J})}$$

$$= \frac{(0.85)(0.2)}{(0.85)(0.2) + (0.15)(0.6)}$$

$$= \frac{17}{26}$$

$$= 0.6538$$

PROBABILITIES MUST BE UPTO 4 d.p

21 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

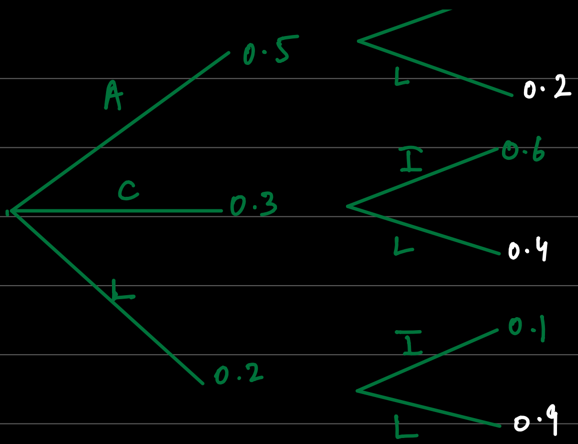
(i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

Second event Result — First event

(ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. [4]

I 0.8

(i) $P(A \text{ and } L) = (0.5)(0.2) = 0.1$



(ii) $P(L \mid I) = \dfrac{P(L \text{ and } I)}{P(I)}$

$= \dfrac{(0.2)(0.1)}{(0.5)(0.8)+(0.3)(0.6)+(0.2)(0.1)}$

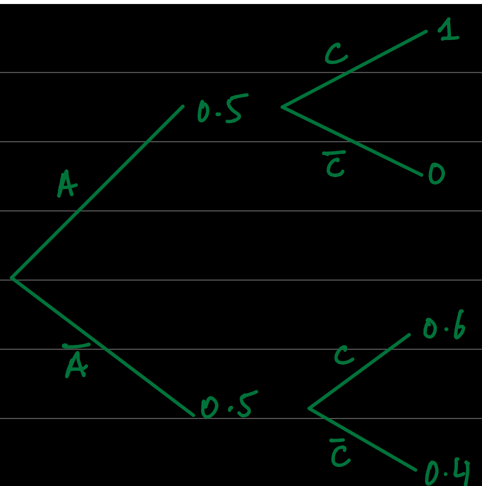
$= \dfrac{1}{30}$

6 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

First event. Second event result.

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]



(i) $P(C) = (0.5)(1) + (0.5)(0.6) = 0.8$

(ii) $P(A \mid C) = \dfrac{P(A \text{ and } C)}{P(C)}$

$= \dfrac{(0.5)(1)}{0.8}$

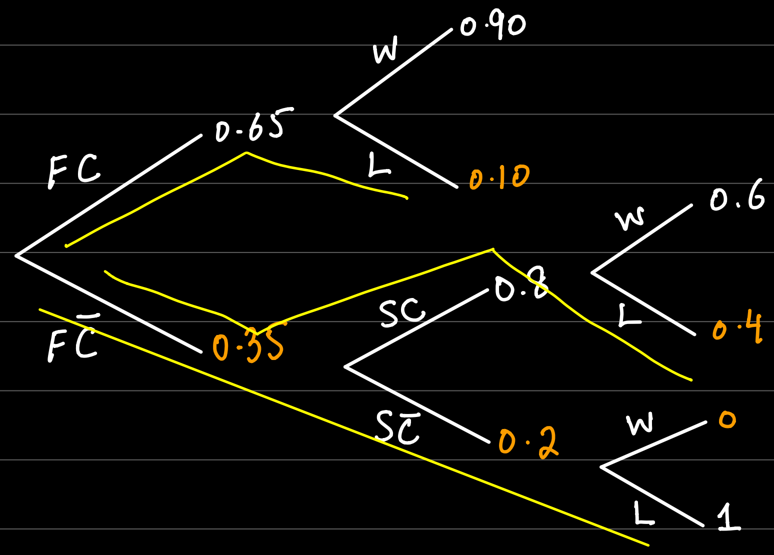
$= \dfrac{5}{8}$

1 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]



(i) $P(L) = \underset{FC\ \ L}{(0.65 \times 0.1)} + \underset{\overline{FC}\ \ SC\ \ L}{(0.35 \times 0.8 \times 0.4)} + \underset{\overline{FC}\ \ \overline{SC}\ \ L}{(0.35 \times 0.2 \times 1)}$

$= 0.247.$

(ii) $P\left(\begin{array}{c|c} \text{First} & \text{Loses} \\ \text{correct} & \text{point} \end{array}\right) = \dfrac{P(FC \text{ and } L)}{P(L)} = \dfrac{(0.65 \times 0.1)}{0.247}$

$= \boxed{0.2632}$

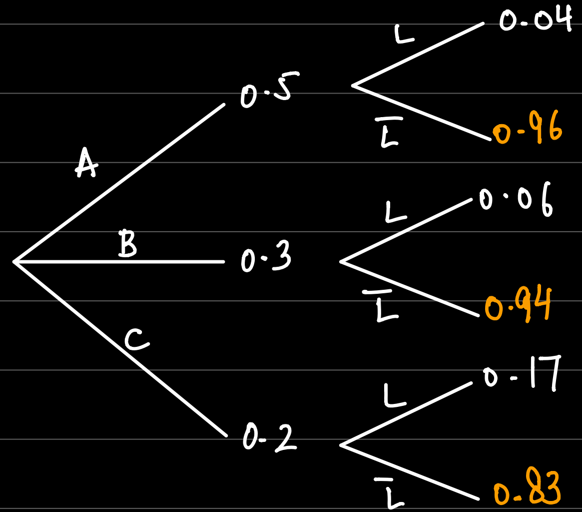
2 When Andrea needs a taxi, she rings one of three taxi companies, A, B or C. 50% of her calls are to taxi company A, 30% to B and 20% to C. A taxi from company A arrives late 4% of the time, a taxi from company B arrives late 6% of the time and a taxi from company C arrives late 17% of the time.

(i) Find the probability that, when Andrea rings for a taxi, it arrives late. [3]

(ii) Given that Andrea's taxi arrives late, find the conditional probability that she rang company B. [3]

Second event

First event.



A and L OR B and L OR C and L

$$P(L) = (0.5)(0.04) + (0.3)(0.06) + (0.2)(0.17)$$

$$P(L) = 0.072.$$

$$P(B \mid L) = \frac{P(B \text{ and } L)}{P(L)} = \frac{(0.3)(0.06)}{0.072}$$

$$= 0.25.$$

8 There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

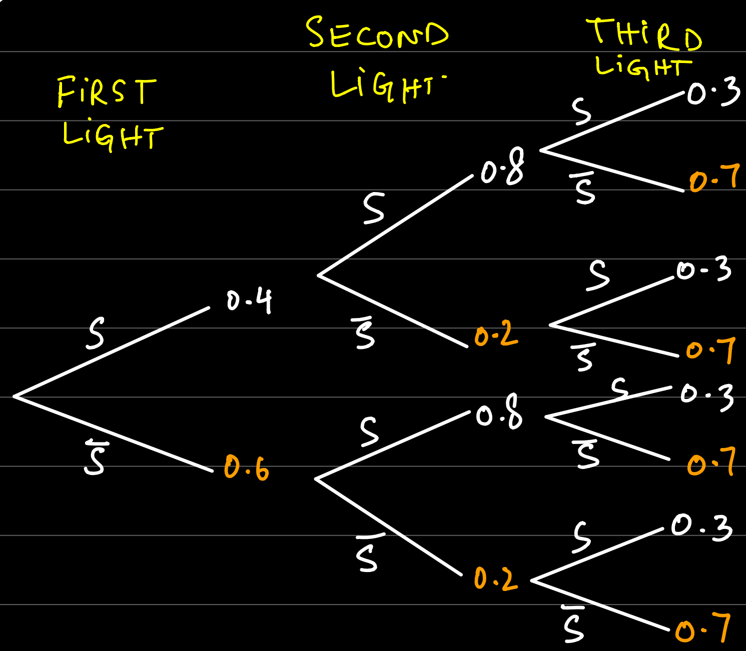
(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]

$\boxed{\text{9+mark}}$

15 min

(i)



(ii) $P(SS\bar{S}) = (0.4)(0.8)(0.7) = 0.224$

$\bar{S}\ S\ S$

(iii) $P(\text{exactly two stops}) = (0.4)(0.8)(0.7) + (0.4)(0.2)(0.3) + (0.6)(0.8)(0.3)$

$= 0.392$

$$P\left(\text{stops at first} \,\middle|\, \text{stops at exactly two}\right) = \frac{P\left(\text{stops at first and stops at exactly two}\right)}{P(\text{exactly two stops})}$$

$$= \frac{\overset{S}{(0.4)}\overset{S}{(0.8)}\overset{\bar{S}}{(0.7)} + \overset{S}{(0.4)}\overset{\bar{S}}{(0.2)}\overset{S}{(0.3)}}{0.392}$$

$$= 0.6327$$

9 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

elephant ride $\frac{6}{10}$, camel ride $\frac{7}{10}$, jungle tractor ride $\frac{8}{10}$.

First event.

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

Second event.

Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

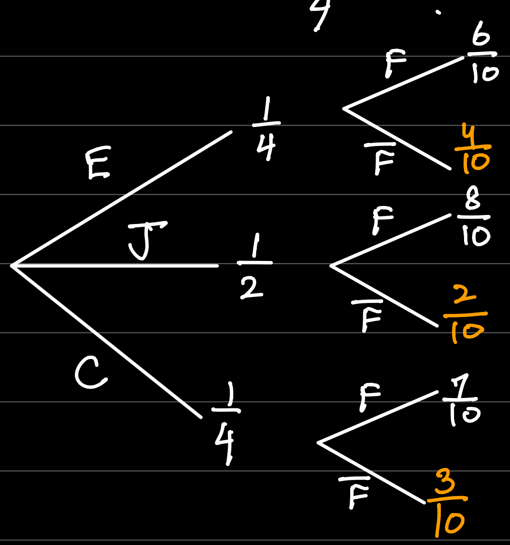
(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

Toss a fair coin twice $= \underbrace{HH}_{\text{Elephant}}, \underbrace{HT, TH}_{\text{Jungle Tractor}}, \underbrace{TT}_{\text{Camel}}$

(i) $P(\text{elephant}) = \frac{1}{4}$

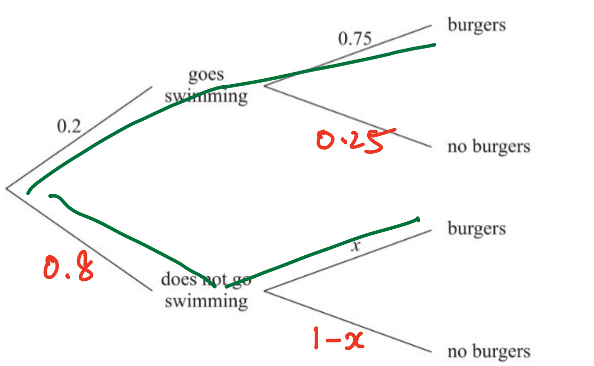
$P(\text{Jungle } T) = \frac{2}{4} = \frac{1}{2}$

$P(\text{Camel}) = \frac{1}{4}$



$P(F) = \left(\frac{1}{4} \times \frac{6}{10}\right) + \left(\frac{1}{2} \times \frac{8}{10}\right) + \left(\frac{1}{4} \times \frac{3}{10}\right) = \frac{29}{40}$

4 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



$\boxed{\text{The probability that Henk has burgers for supper on any day is 0.5.}}$ $P(B) = 0.5$

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

(i) $P(B) = (0.2)(0.75) + (0.8)(x)$

$0.5 = (0.2)(0.75) + (0.8)(x)$

$x = 0.4375$

(ii) $P(S \mid B) = \dfrac{P(S \text{ and } B)}{P(B)}$

$= \dfrac{(0.2)(0.75)}{0.5}$

$= \dfrac{3}{10}$

$= 0.3$

V. IMP

LISTING IS A PROPER TECHNIQUE IN A LEVES (S1)

15 A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

- If the spinner lands on an **even-numbered** side, Raj **multiplies** the two numbers showing on the dice to get his score.
- If the spinner lands on an **odd-numbered** side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side. [6]

SPINNER : $P(E) = \frac{2}{5}$ 2, 4 | $P(O) = \frac{3}{5}$ 1, 3, 5

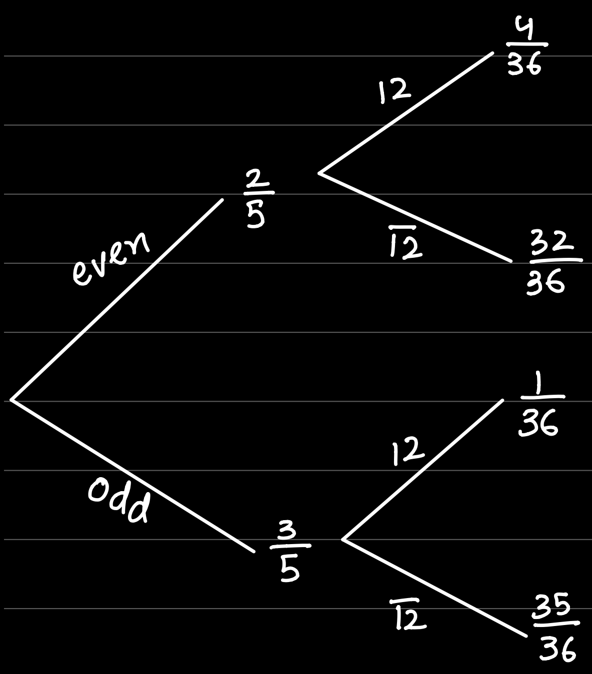
2DICE : MULTIPLY | ADD

(3, 4)
(4, 3)
(2, 6)
(6, 2)

$\boxed{\frac{4}{36}} = P(12)$

(6, 6) $P(12) = \frac{1}{36}$

Total outcomes when we throw two dices = 36.



$$P(E \mid 12) = \frac{P(E \text{ and } 12)}{P(12)}$$

$$= \frac{\left(\frac{2}{5} \times \frac{4}{36}\right)}{\left(\frac{2}{5} \times \frac{4}{36}\right) + \left(\frac{3}{5} \times \frac{1}{36}\right)}$$

$$= \frac{8}{11}$$

Tree diagrams are usually very favorable if events are few and are one after another.

14 Three friends, Rick, Brenda and Ali, go to a football match but forget to say which entrance to the ground they will meet at. There are four entrances, A, B, C and D. Each friend chooses an entrance independently.

- The probability that Rick chooses entrance A is $\frac{1}{3}$. The probabilities that he chooses entrances B, C or D are all equal.
- Brenda is equally likely to choose any of the four entrances.
- The probability that Ali chooses entrance C is $\frac{2}{7}$ and the probability that he chooses entrance D is $\frac{3}{5}$. The probabilities that he chooses the other two entrances are equal.

(i) Find the probability that at least 2 friends will choose entrance B. [4]

(ii) Find the probability that the three friends will all choose the same entrance. [4]

	Rick	Brenda	Ali
A	$\frac{1}{3} = \frac{3}{9}$	$\frac{1}{4}$	$\frac{2}{35}$
B	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{2}{35}$
C	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{2}{7}$
D	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{3}{5}$

(i) At least two friends choose B :

Two friends go to B $\boxed{\text{OR}}$ 3 friends go to B.

$$R_B \overset{\text{and}}{\uparrow} B_B \overset{\text{and}}{\uparrow} A_{\overline{B}} = \left(\frac{2}{9} \times \frac{1}{4} \times \frac{33}{35}\right) = \frac{11}{210}$$

$\boxed{\text{OR}}$

$$R_B \stackrel{\text{and}}{\ } B_{\bar{B}} \stackrel{\text{and}}{\ } A_B = \left(\frac{2}{9} \times \frac{3}{4} \times \frac{2}{35}\right) = \cancel{\frac{1}{65}}\ \frac{1}{105}$$

$\boxed{\text{OR}}$

$$R_{\bar{B}} \stackrel{\text{and}}{\ } B_B \stackrel{\text{and}}{\ } A_B = \left(\frac{7}{9} \times \frac{1}{4} \times \frac{2}{35}\right) = \frac{1}{90}$$

$\boxed{\text{OR}}$ $$R_B\ B_B\ A_B = \left(\frac{2}{9} \times \frac{1}{4} \times \frac{2}{35}\right) = \frac{1}{315}$$

$$\frac{8}{105}$$

(11)

	Rick	Brenda	Ali
A	$\frac{1}{3} = \frac{3}{9}$	$\frac{1}{4}$	$\frac{2}{35}$
B	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{2}{35}$
C	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{2}{7}$
D	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{3}{5}$

$$\text{All}(A) = \frac{1}{3} \times \frac{1}{4} \times \frac{2}{35} =$$

$\boxed{\text{OR}}$

$$\text{All}(B) = \frac{2}{9} \times \frac{1}{4} \times \frac{2}{35} =$$

$\boxed{\text{OR}}$

$$\text{All}(C) = \frac{2}{9} \times \frac{1}{4} \times \frac{2}{7} =$$

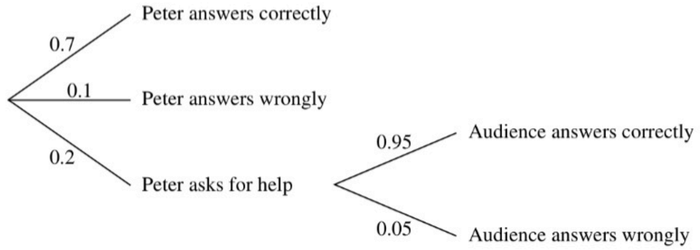
$\boxed{\text{OR}}$

$$\text{All}(D) = \frac{2}{9} \times \frac{1}{4} \times \frac{3}{5} =$$

11 In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.



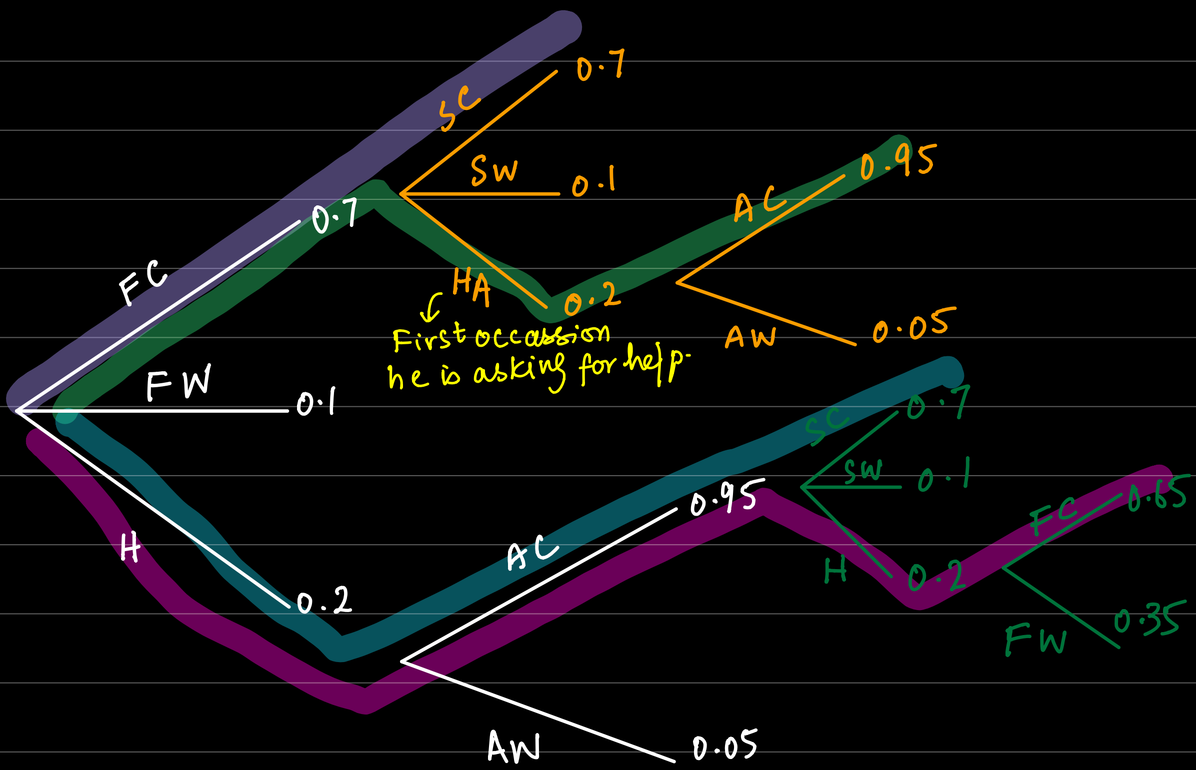
(i) Show that the probability that the first question is answered correctly is 0.89. [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

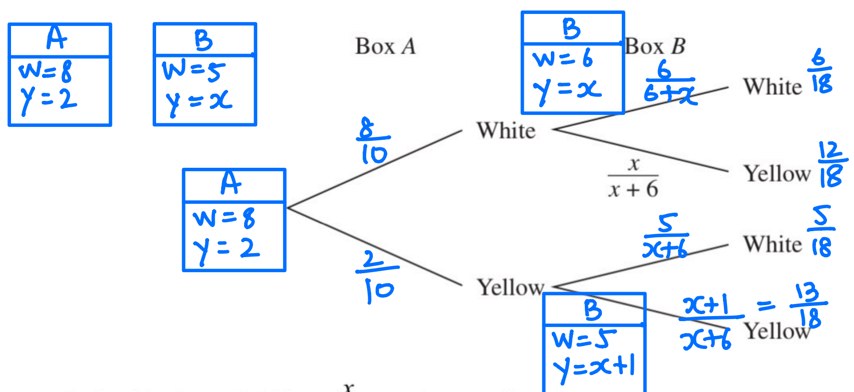
(ii) Find the probability that the first two questions are both answered correctly. [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience. [3]

(i) $P(C) = 0.7 + (0.2)(0.95) = 0.89$



23 Box A contains 8 white balls and 2 yellow balls. Box B contains 5 white balls and x yellow balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.



(i) Justify the probability $\frac{x}{x+6}$ on the tree diagram. [1]

(ii) Copy and complete the tree diagram. [4]

(iii) If the ball chosen from box A is white then the probability that the ball chosen from box B is also white is $\frac{1}{3}$. Show that the value of x is 12. [2]

(iv) Given that the ball chosen from box B is yellow, find the conditional probability that the ball chosen from box A was yellow. [4]

(iii) $\frac{6}{\quad} = \frac{1}{\quad}$ | (iv) $P(Y_A \mid Y_B) = \frac{P(Y_A \text{ and } Y_B)}{\quad}$

$6+x \qquad 3$

$18 = 6 + x$

$x = 12$

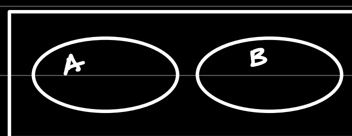
$P(Y_B)$

$$= \frac{\frac{2}{10} \times \frac{13}{18}}{\left(\frac{8}{10} \times \frac{12}{18}\right) + \left(\frac{2}{10} \times \frac{13}{18}\right)}$$

$=$

MUTUALLY EXCLUSIVE EVENTS (1-2 Marks)

Events that can never happen together.



$\boxed{A \cap B = 0}$

eg: Toss a coin. H T

Toss Two coins HH TT HT TH

EXAM: You have to explore and comment if the two events can happen together or not. No calculation is required.

INDEPENDENT EVENTS (2 — 5 Marks)

Those events which satisfy this equation.

$$P(A \text{ and } B) = P(A) \times P(B)$$

EXAM: Calculate all three of these probabilities and put them in this equation.

12 Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.

R : both of the numbers are greater than 8.

(i) Find P(Q). [2]

(ii) Find P(R). [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

Total outcomes = $12 \times 12 = 144$.

$\boxed{Q}$ $\left.\begin{matrix} (2,12) & (3,8) & (4,6) \\ (12,2) & (8,3) & (6,4) \end{matrix}\right\}$ $P(Q) = \frac{6}{144} = \frac{1}{24}$

$\boxed{R}$ $\left.\begin{matrix} (9,9) & (10,9) & (11,9) & (12,9) \\ (9,10) & (10,10) & (11,10) & (12,10) \\ (9,11) & (10,11) & (11,11) & (12,11) \\ (9,12) & (10,12) & (11,12) & (12,12) \end{matrix}\right\}$ $P(R) = \frac{16}{144} = \frac{1}{9}$

(iii) yes Q and R are mutually exclusive since no combination of numbers greater than 8 can give product of 24.

(iv) $P(Q \text{ and } R) = P(Q) \times P(R)$

together

$0 \neq \frac{1}{24} \times \frac{1}{9}$

Not independent.